9.4 Tests for Convergence

geometric series $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 \ldots$

if $|r| < 1$ the series converges $= \frac{a}{1-r}$

if $|r| \geq 1$ the series diverges

$\sum_{n=1}^{\infty} 2\left(-\frac{1}{5}\right)^{n-1}$ $r = -\frac{1}{5}$ $\left|-\frac{1}{5}\right| < 1$ converges to $= \frac{2}{1 - -\frac{1}{5}}$

$\sum_{n=1}^{\infty} \left(\frac{5}{2}\right)^{n-1}$ $r = \frac{5}{2} > 1$ series diverges

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ratio test

if $\sum_{n=1}^{\infty} a_n$ pos series

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$

works well for $n$ - exponent or $n!$

if $L < 1$ the series converges

if $L > 1$ the series diverges

if $L = 1$ the test is inconclusive

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$$\sum_{n=0}^{\infty} \frac{3^n}{5^n+1}$$

$$\lim_{n\to\infty} \frac{\dfrac{3^{n+1}}{5^{n+1}+1}}{\dfrac{3^n}{5^n+1}} \quad \begin{matrix} a_{n+1} \\ a_n \end{matrix}$$

$$\lim_{n\to\infty} \frac{\cancel{3^{n+1}}}{5^{n+1}\cancel{+1}} \cdot \frac{5^n \cancel{+1}}{\cancel{3^n}} = \frac{3}{5} = L$$

series converges by the ratio test

$$\sum_{n=0}^{\infty} \frac{3^n}{5^n+1} = \frac{1}{2} + \frac{3}{6} + \frac{3^2}{5^2+1} \cdots$$

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$$\sum_{n=0}^{\infty} \frac{2^n}{n!}$$

converges by ratio test

$L = 0 < 1$

$$\lim_{n\to\infty} \frac{\dfrac{2^{n+1}}{(n+1)!}}{\dfrac{2^n}{n!}} = \lim_{n\to\infty} \frac{2^{\cancel{n+1}}}{(n+1)\cancel{!}} \cdot \frac{\cancel{n!}}{\cancel{2^n}}$$

~~$n(n-1)(n-2)\ldots1$~~

$(n+1)\cancel{n(n-1)(n-2)\ldots1}$

$$= \lim_{n\to\infty} \frac{2}{n+1} = 0$$

$L = 0$

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$n^{th}$ term test

seq does not approach 0

$\sum a_n$ if $a_n \not\to 0$ then $\sum a_n$ diverges

$$\sum_{n=1}^{\infty} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$$

seq $\to 1$ $a_n \to 1$

so $a_n \not\to 0$

Series diverges by the $n^{th}$ term test

If $a_n \to 0$, test is inconclusive

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Comparison test

$\sum a_n$ no neg terms

If $a_n \le c_n$ & $\sum c_n$ converges

then $\sum a_n$ converges

If $a_n \ge d_n$ & $\sum d_n$ diverges

$\sum a_n$ diverges

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$$\sum_{n=0}^{\infty} \frac{1}{(n!)^2} \leq \sum_{n=0}^{\infty} \frac{1}{n!} \leftarrow \text{converges by ratio test}$$

converges by ratio

ratio test $\lim_{n\to\infty} \frac{\frac{1}{(n+1)\cancel{n!}}}{\frac{1}{\cancel{n!}}} \cdot \frac{\cancel{n!}}{1} \quad L = 0 < 1$

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absolute convergence test

If $\sum |a_n|$ converges then $\sum a_n$ also converges

could be negative

$$\sum_{n=0}^{\infty} \frac{(\sin x)^n}{n!} \text{ converges absolutely}$$

$$\sum_{n=0}^{\infty} \frac{|\sin x|^n}{n!} \leq \sum_{n=0}^{\infty} \frac{1}{n!} \quad \text{converges by ratio test}$$

converges by comparison

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